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Seiberg-Witten curves for elliptic models [1]

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Abstract

Four-dimensional $\mathcal{N}$=2 gauge theories may be obtained from configurations of D-branes in type IIA string theory. Unitary gauge theories with two-index representations, and orthogonal and symplectic gauge theories, are constructed from configurations containing orientifold planes. Models with two orientifold planes imply a compact dimension, and correspond to elliptic models. Lifting these configurations to M-theory allows one to derive the Seiberg-Witten curves for these gauge theories. We describe how the Seiberg-Witten curves, necessarily of infinite order, are obtained for these elliptic models. These curves are used to calculate the instanton expansion of the prepotential; we explicitly find the one-instanton prepotential for all the elliptic models considered.

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1. Introduction

In the Seiberg-Witten approach to four-dimensional $\mathcal{N} = 2$ supersymmetric gauge theories [1], one begins by identifying an algebraic curve and meromorphic differential specific to the gauge group and matter content of the theory. One then calculates the periods of this differential, and integrates the result to obtain the exact low-energy prepotential for the gauge theory. The perturbative and instanton contributions to the prepotential may then be compared with results obtained directly from the microscopic Lagrangian of the gauge theory. General methods were presented in Refs. [2] for computing the prepotential for gauge theories with hyperelliptic curves. For theories with non-hyperelliptic curves, a systematic approximation scheme for calculating the instanton expansion of the prepotential was developed in Refs. [3]-[7] and is described in the talk by H. J. Schnitzer at this Workshop [8].

M-theory provides a systematic means of deriving Seiberg-Witten curves [9]. One identifies a type IIA brane configuration that gives rise to the four-dimensional gauge theory of interest, and then lifts this to an M5 brane configuration; the world-volume of the M5 brane contains the Seiberg-Witten curve as a factor.

Our goal in this talk is to explain how to derive the one-instanton prepotentials for the class of elliptic models with a simple gauge group [10]. In sect. 2, we describe the construction of type IIA brane configurations that correspond to these four-dimensional $\mathcal{N} = 2$ gauge theories. We explain in sect. 3 how the lift to M-theory may be used to obtain the Seiberg-Witten curves for these theories. In sect. 4, the quartic truncation of the SW curve is used to obtain explicit expressions for the one-instanton prepotentials for each of these theories.

2. Type IIA brane configurations and four-dimensional gauge theory

We begin by briefly reviewing type IIA brane configurations associated with various four-dimensional $\mathcal{N} = 2$ gauge theories; see Ref. [11] for a review with references. A typical brane configuration, shown in fig. 1, contains a number of NS 5-branes, extended in the 012345 directions, located at the same point in the 789 directions, and having distinct values of x_6. The horizontal direction in the figure corresponds to x_6, the vertical direction to $v = x_4 + ix_5$, with the remaining directions suppressed. The NS 5-branes are connected by D4-branes, extended in the 01236 directions, but of finite length along x_6.

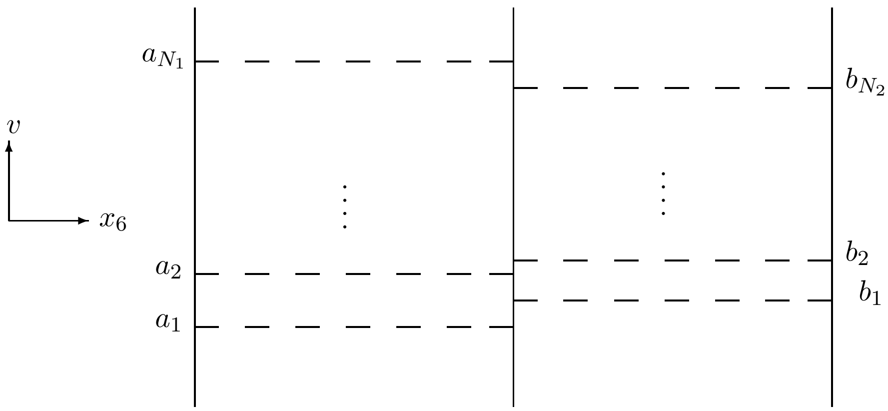

Figure 1

The brane configuration in fig. 1 gives rise to an $\mathcal{N} = 2$ $\mathrm{SU}(N_1) \times \mathrm{SU}(N_2)$ gauge theory with a matter hypermultiplet in the bifundamental representation [9]. The first two NS 5-branes are connected by N_1 D4-branes; strings extending between the latter give rise to the adjoint vector multiplet of the gauge group $\mathrm{SU}(N_1)$. Strings extending between the N_2 D4-branes connecting the last two NS 5-branes yield the adjoint vector multiplet of the gauge group $\mathrm{SU}(N_2)$. Finally, strings extending between the D4 branes connecting the first two NS 5-branes and the D4 branes connecting the last two NS 5-branes give rise to the hypermultiplet in the $(\overline{\square}, \square)$ representation of $\mathrm{SU}(N_1) \times \mathrm{SU}(N_2)$.

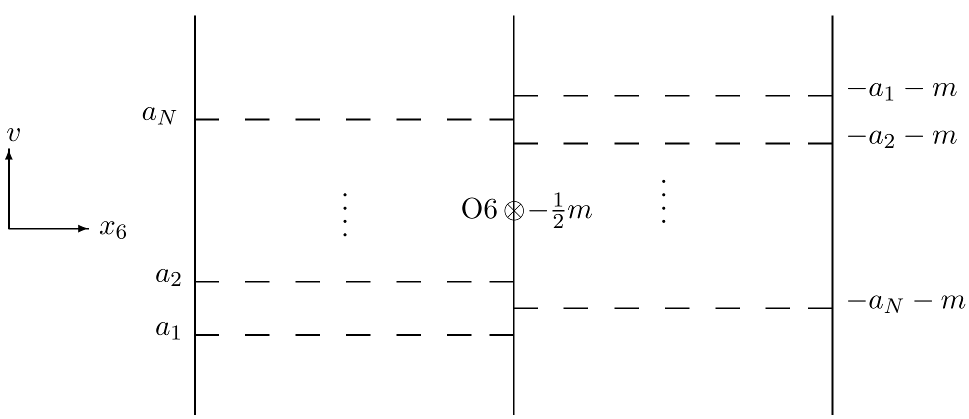

Figure 2

Figure 2 contains an orientifold 6-plane extending in the 0123789 directions, and intersecting the central NS 5-brane. The orientifold 6-plane can have either $+4$ or -4 units of 6-brane charge, and is designated O6$^+$ or O6$^-$ respectively. The O6 plane identifies the points $(x_6, v) \sim$

$(-x_6, -v - m)$ in the directions transverse to it. In terms of the gauge theory, the orientifold identifies the two factors of the gauge group $\mathrm{SU}(N) \times \mathrm{SU}(N)$, and projects the bifundamental representation onto either the symmetric $\Box\!\Box$ representation (O6$^+$) or the antisymmetric $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ representation (O6$^-$) [12].

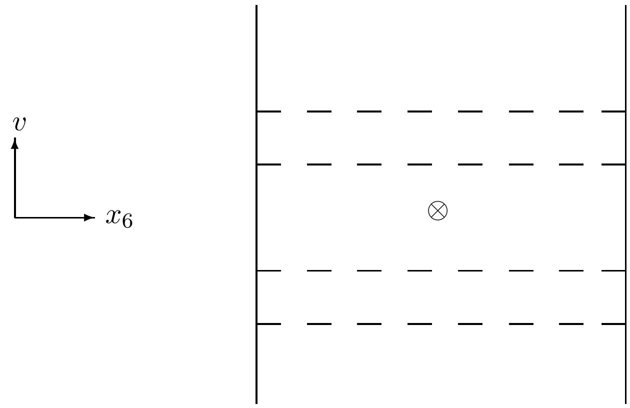

Figure 3

In fig. 3, the orientifold plane is located midway between the NS 5-branes, and serves to project out some of the states of the adjoint representation of the SU($2N$) gauge group, leaving either SO($2N$) for O6$^+$ or Sp($2N$) for O6$^-$ [12].

Most asymptotically-free $\mathcal{N} = 2$ gauge theories can be obtained from a variant of the brane configurations just described, either with or without an orientifold plane. However, SU(N) gauge theory with *two* antisymmetric hypermultiplets apparently requires a configuration with at least two O6$^-$ planes, as described in Ref. [7] and in the talk of H. J. Schnitzer at this Workshop [8]. These two O6$^-$ planes, moreover, generate an infinite number of O6$^-$ planes and NS 5-branes, equally spaced in the x_6 direction. The corresponding SW curve would be of infinite order.

Alternatively, we may observe that a pair of reflections through different points generates a translation, so the brane configuration must in fact be periodic in the x_6 direction. A second compactified direction, x_{10}, emerges in M-theory, so that the lifted M5 configuration lives on a torus. We are thus naturally led to a discussion of elliptic models [13, 9, 14]. The infinite order SW curve may be regarded as an elliptic curve written on the covering space (see, e.g., ref. [15]).

We now describe the class of elliptic brane configurations, containing a pair of O6$^\pm$ planes, that give rise to four-dimensional gauge theories with simple gauge groups and vanishing beta function [14]. The latter condition requires that the total 6-brane charge vanish, so that configurations with two O6$^-$ planes also contain four D6 branes (plus mirrors) parallel to the O6 planes; configurations with O6$^+$ and O6$^-$ require no D6 branes.

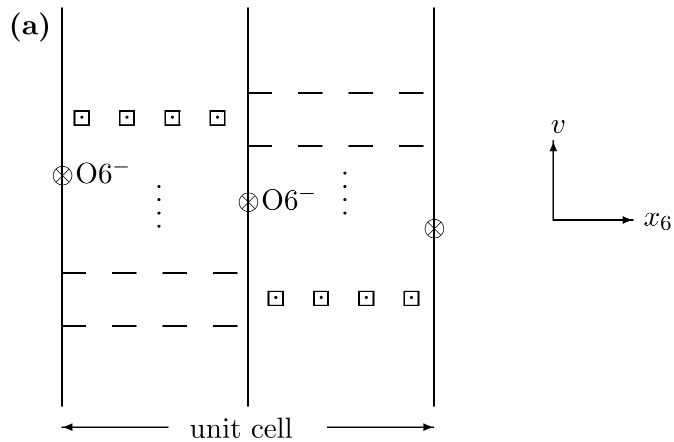

Figure 4

Figures 4 and 5 show only the unit cell of the periodic configurations; the left-and right-most NS 5-branes are to be identified (with a possible shift in the v direction). The corresponding gauge theories may be identified using the rules described above. The configurations in fig. 4 contain two NS 5-branes per unit cell. Figure 4(a) corresponds to SU(N) gauge theory with two antisymmetric hypermultiplets and four fundamental hypermultiplets, whose degrees of freedom arise from strings stretched between the D4- and D6-branes. Figure 4(b) corresponds to SU(N) gauge theory with an antisymmetric and a symmetric hypermultiplet.

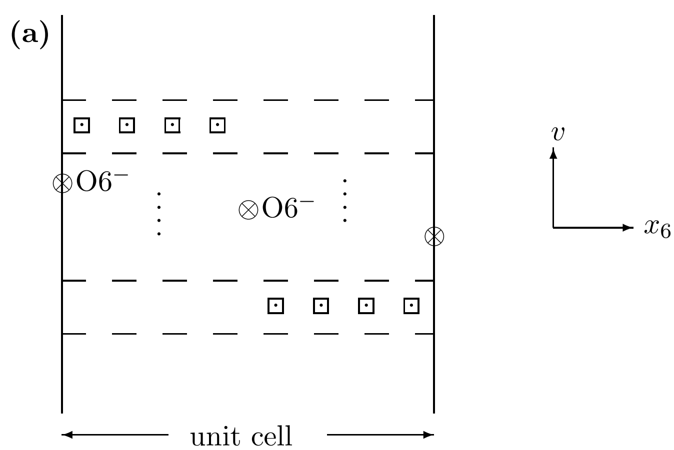

Figure 5

The configurations in fig. 5 contain only one NS 5-brane per unit cell. Figure 5(a) corresponds to Sp($2N$) gauge theory with an antisymmetric hypermultiplet and four fundamental hypermultiplets. Figure 5(b) corresponds to Sp($2N$) gauge theory with an adjoint hypermultiplet (O6$^+$ on the NS 5-brane), or SO($2N$) gauge theory with an adjoint hypermultiplet (O6$^-$ on the NS 5-brane).

Finally, a periodic configuration without orientifold planes and with one NS 5-brane per unit cell corresponds to SU(N) gauge theory with an adjoint hypermultiplet [9].

3. M theory and Seiberg-Witten curves

In the strong-coupling limit, type IIA string theory goes over to eleven-dimensional M-theory with an additional periodic coordinate x_{10} (with period R); the brane configurations described in the previous section are "lifted" to M5-brane configurations [9]. The M5-brane world-volume is $\mathbb{R}^4 \times \Sigma$ where $\mathbb{R}^4$ spans the 0123 directions, and Σ is a two-dimensional submanifold of $Q \sim \mathbb{C}^2 = (v, t)$, where $v = x_4 + ix_5$ and $t = \exp\left[-(x_6 + ix_{10})/R\right]$. The M5-brane is located at a point in the remaining 789 directions. $\Sigma \subset Q$ can be written as an algebraic curve, which is none other than the Seiberg-Witten curve of the corresponding four-dimensional gauge theory.

The M5-brane curve Σ corresponding to the IIA configuration shown in fig. 1 is [9, 16]

$$t^3 - \prod_{i=1}^{N_1}(v - a_i)\, t^2 + \Lambda^{2N_1 - N_2} \prod_{i=1}^{N_2}(v - b_i)\, t - \Lambda^{3N_1} = 0. \tag{3.1}$$

The features of this curve can be understood directly in terms of the classical IIA picture. Holding v fixed, eq. (3.1) has three solutions for t; these correspond to the positions of the NS 5-branes. The coefficients of the various powers of t vanish at the positions of the D4-branes between adjacent NS 5-branes. Since there are no D4 branes to the left and right of the NS 5-branes, the first and last terms of the curve have constant coefficients. The curve (3.1) is indeed the SW curve of the SU(N_1) $\times$ SU(N_2) gauge theory associated with this IIA configuration.

The M-theory geometry corresponding to a type IIA configuration involving an orientifold plane is more complicated. For a single O6$^-$ plane, as in fig. 2, the M-theory background is an Atiyah-Hitchin space [17]. This may be described in terms of a submanifold $\tilde{Q}$ of $\mathbb{C}^3 = (v, t_L, t_R)$. Far from the orientifold plane, $\tilde{Q}$ is given by [12]

$$t_L t_R^{-1} = \frac{\Lambda^{2N+4}}{\left(v + \frac{1}{2}m\right)^4} \tag{3.2}$$

and is invariant under

$$v \to -v - m, \qquad t_L \to t_R^{-1}. \tag{3.3}$$

In the region far to the left of the orientifold plane ($x_6 \to -\infty$), the variable $t_L \to t$, whereas in the region far to the right of the orientifold plane ($x_6 \to \infty$), the variable $t_R \to t$.

The M5-brane configuration corresponding to the type IIA configuration shown in fig. 2 is given by $\mathbb{R}^4 \times \Sigma$, where now Σ is an algebraic curve embedded in $\tilde{Q}$,

$$t_L^3 + \prod_{i=1}^{N} (v - a_i)\, t_L^2 + A(v)\, t_L + B(v) = 0. \tag{3.4}$$

Since t_L corresponds to t to the left of the orientifold plane, the coefficients of the first two (but not the last two) terms correspond to the positions of the D4-branes in that region of fig. 2. The curve Σ must be invariant under (3.3), and so may be rewritten as

$$B(-v-m)\, t_R^3 + A(-v-m)\, t_R^2 + \prod_{i=1}^{N} (-v-m-a_i)\, t_R + 1 = 0 \tag{3.5}$$

where now the coefficients of the last two terms correspond to the positions of the D4-branes to the right of the orientifold plane in fig. 2. Using eq. (3.2), we may rewrite eq. (3.5) in terms of t_L; equating the result with eq. (3.4), we finally obtain

$$t_L^3 + \prod_{i=1}^{N} (v - a_i)\, t_L^2 + \frac{\Lambda^{N+2} \prod_{i=1}^{N} (-v - a_i - m)}{(v + \frac{1}{2}m)^2}\, t_L + \frac{\Lambda^{3N+6}}{(v + \frac{1}{2}m)^6} = 0. \tag{3.6}$$

We have used eq. (3.2), which is valid only far from the orientifold; consequently, eq. (3.6) only gives the leading terms (in powers of Λ) of the curve. The subleading terms may be determined by a more careful consideration of the Atiyah-Hitchin space $\tilde{Q}$ [12]. Including the subleading terms, and defining $y = t_L/(v + \frac{1}{2}m)^2$, we obtain the curve

$$\begin{aligned} y^3 \quad &+ \quad y^2 \left[(v + \tfrac{1}{2}m)^2 \prod_{i=1}^{N} (v - a_i) + 3\Lambda^{N+2} \right] \\ &+ \quad y\Lambda^{N+2} \left[(v + \tfrac{1}{2}m)^2 \prod_{i=1}^{N} (-v - a_i - m) + 3\Lambda^{N+2} \right] \; + \; \Lambda^{3N+6} \; = \; 0, \end{aligned} \tag{3.7}$$

which is the form of the curve given by Landsteiner and Lopez [12]. From this curve, the one-instanton propotential may be calculated [3].

After these warm-ups, we turn to the calculation of the SW curves for the elliptic IIA configurations shown in figs. 4 and 5. For concreteness, we focus on the configuration in fig. 4(b), which gives rise to the four-dimensional $\mathcal{N} = 2$ supersymmetric SU(N) gauge theory with hypermultiplets in the symmetric $\square\!\square$ and antisymmetric $\begin{smallmatrix}\square\\[-2pt]\square\end{smallmatrix}$ representations, but the other cases are analogous.

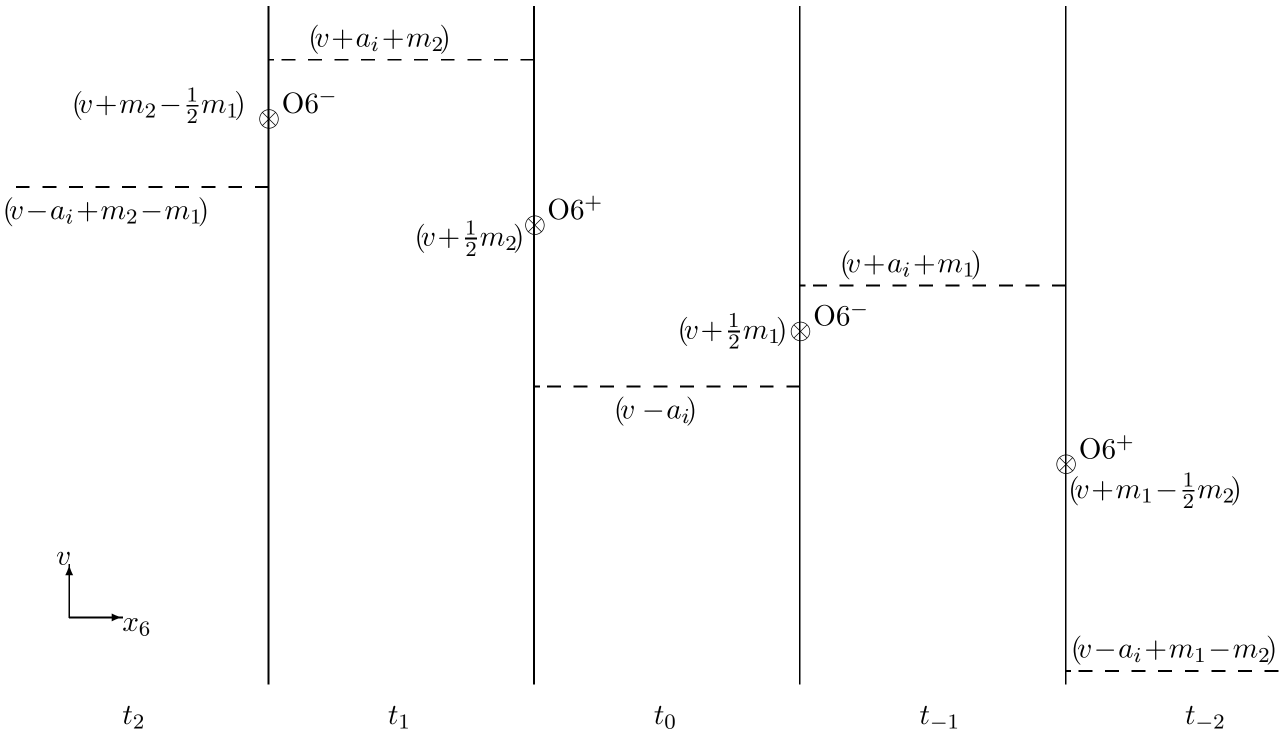

Figure 6

Figure 6 shows a piece of the IIA configuration on the covering space. The displacements of the O6 planes in the v direction correspond to the masses of the hypermultiplets; here m_1 (m_2) is the mass of the antisymmetric (symmetric) hypermultiplet.

We would like to derive the M5-brane curve $\mathbb{R}^4 \times \Sigma$ that corresponds to the configuration in fig. 6. First we must describe the M-theory geometry in which Σ is embedded. Because of the presence of orientifold planes, we introduce an infinite set of "local" variables t_n. In the region between any pair of orientifold planes, the corresponding variable t_n shown in fig. 6 corresponds to $t = \exp[-(x_6 + ix_{10})/R]$. The two variables t_0 and t_{-1} adjacent to the O6$^-$ plane at $v = -\frac{1}{2}m_1$ are related, far from the plane, by

$$t_0 t_{-1}^{-1} = \frac{q^{1/2}}{\left(v + \frac{1}{2}m_1\right)^4} \tag{3.8}$$

as in eq. (3.2), where for elliptic models the dimensionless parameter q replaces the scale factor Λ. The two variables t_0 and t_1 adjacent to the O6$^+$ plane at $v = -\frac{1}{2}m_2$ are related, far from the plane, by [12]

$$t_1 t_0^{-1} = q^{1/2}\left(v + \tfrac{1}{2}m_2\right)^4. \qquad (3.9)$$

The pairs of variables adjacent to each of the other O6 planes are related by analogous "transition functions."

Let us write the (leading terms of the) infinite order curve Σ as

$$\cdots \;+\; qP_2(v)t_0^2 \;+\; q^{1/4}P_1(v)t_0 \;+\; P_0(v) \;+\; q^{1/4}P_{-1}(v)t_0^{-1} \;+\; qP_{-2}(v)t_0^{-2} \;+\; \cdots \;=\; 0 \qquad (3.10)$$

in terms of the local variable t_0. In the central region of fig. 6, t_0 corresponds to t, so the coefficient $P_0(v)$ must vanish at the positions $v = a_i$ of the N D4 branes in that region, i.e. $P_0(v) = \prod_{i=1}^{N}(v - a_i)$. To determine the other coefficients, $P_n(v)$, we must use the invariance of the curve induced by the orientifold planes, together with the "transition functions" (3.8) and (3.9). First, requiring that the curve be invariant under

$$v \to -v - m_1, \qquad t_0 \to t_{-1}^{-1} \qquad (3.11)$$

we may rewrite eq. (3.10) as

$$\begin{aligned} \cdots \;&+ q^{1/4}P_1(-v-m_1)\,t_{-1}^{-1} \;+\; P_0(-v-m_1) \\ &+ q^{1/4}P_{-1}(-v-m_1)\,t_{-1} \;+\; qP_{-2}(-v-m_1)\,t_{-1}^2 + \cdots = 0. \end{aligned} \qquad (3.12)$$

We then use eq. (3.8) to obtain

$$\begin{aligned} \cdots \;&+ q^{1/4}(v+\tfrac{1}{2}m_1)^6 P_{-2}(-v-m_1)t_0 \;+\; (v+\tfrac{1}{2}m_1)^2 P_{-1}(-v-m_1) \\ &+ q^{1/4}(v+\tfrac{1}{2}m_1)^{-2}P_0(-v-m_1)t_0^{-1} \;+\; q(v+\tfrac{1}{2}m_1)^{-6}P_1(-v-m_1)t_0^{-2} + \cdots = 0 \end{aligned} \qquad (3.13)$$

Equating this with eq. (3.10), we find

$$\begin{aligned} P_{-1}(v) &= (v+\tfrac{1}{2}m_1)^{-2}P_0(-v-m_1), \\ P_{-2}(v) &= (v+\tfrac{1}{2}m_1)^{-6}P_1(-v-m_1), \\ &\vdots \end{aligned} \qquad (3.14)$$

Next, we require that the curve be invariant under

$$v \to -v - m_2, \qquad t_0 \to t_1^{-1}. \qquad (3.15)$$

This, together with eq. (3.9), generates a set of relations similar to eq. (3.15). The two reflections (3.11) and (3.15) generate the entire invariance group, and so are sufficient to give us (the leading terms of) the entire set of coefficients

$$
\begin{aligned}
& \vdots \\
P_2(v) &= (v+\tfrac{1}{2}m_2)^6\,(v+m_2-\tfrac{1}{2}m_1)^{-2}\prod_{i=1}^{N}(v-a_i+m_2-m_1), \\
P_1(v) &= (v+\tfrac{1}{2}m_2)^2\prod_{i=1}^{N}(-v-a_i-m_2), \\
P_0(v) &= \prod_{i=1}^{N}(v-a_i), \\
P_{-1}(v) &= (v+\tfrac{1}{2}m_1)^{-2}\prod_{i=1}^{N}(-v-a_i-m_1), \\
P_{-2}(v) &= (v+\tfrac{1}{2}m_1)^{-6}\,(v+m_1-\tfrac{1}{2}m_2)^2\prod_{i=1}^{N}(v-a_i+m_1-m_2), \\
& \vdots
\end{aligned}
\tag{3.16}
$$

This is equivalent to the curve given in ref. [10], up to overall multiplication by $F(v)$ and redefinition $t_0 = G(v)t$, where $F(v)$ and $G(v)$ are rational functions of v and the hypermultiplet masses. The prepotential derived from these curves is independent of $F(v)$ and $G(v)$.

In Refs. [7, 10], SW curves are given for all the other elliptic models discussed in the previous section.

In the limit $m_1 \to m_2$ (i.e., vanishing global mass), there are no subleading terms, and the curve (3.10) for $\mathrm{SU}(N) + \square\!\square + \begin{smallmatrix}\square\\[-2pt]\square\end{smallmatrix}$ reduces, upon change of variable $t = t_0(v+\frac{1}{2}m)^2$, to

$$
\begin{aligned}
0 &= \sum_{n\,\mathrm{even}} q^{n^2/4}\,t^n\prod_{i=1}^{N}(v-a_i) + \sum_{n\,\mathrm{odd}} q^{n^2/4}\,t^n\prod_{i=1}^{N}(-v-a_i-m) \\
&= \theta_3\left(\frac{z}{\omega_1}|2\tau\right)\prod_{i=1}^{N}(v-a_i) + \theta_2\left(\frac{z}{\omega_1}|2\tau\right)\prod_{i=1}^{N}(-v-a_i-m),
\end{aligned}
\tag{3.17}
$$

where $q=\exp(2\pi i\tau)$, $t=\exp(-i\pi z/\omega_1)$, and $\theta_2(\nu|\tau)$, $\theta_3(\nu|\tau)$ are Jacobi theta functions. In ref. [10], we have shown that eq. (3.17) is equivalent to the curve for this theory given by Uranga [14].

4. One-instanton prepotential

Although we have obtained an infinite order curve for the $\mathrm{SU}(N) + \square\!\square + \begin{smallmatrix}\square\\ \square\end{smallmatrix}$ theory, the one-instanton ($\mathcal{O}(q)$) prepotential for this theory may be extracted [6, 7] from the quartic truncation of this curve consisting of just those five terms shown explicitly in eq. (3.17). Define

$$S(v) = \frac{P_1(v)P_{-1}(v)}{P_0(v)^2}. \tag{4.1}$$

For this theory, $S(v)$ has quadratic poles at $v = a_k$ and $v = -\frac{1}{2}m_1$. At these poles, we define the residue functions $S_k(v)$ and $S_{m_1}(v)$ by

$$S(v) \;=\; \frac{S_k(v)}{(v-a_k)^2} \;=\; \frac{S_{m_1}(v)}{(v+\frac{1}{2}m_1)^2}. \tag{4.2}$$

It may be shown that the one-instanton prepotential is given by

$$2\pi i \mathcal{F}_{1-\mathrm{inst}} = \sum_{k=1}^{N} S_k(a_k) - 2S_{m_1}(-\tfrac{1}{2}m_1). \tag{4.3}$$

Although $S(v)$ and therefore eq. (4.3) depend explicitly only on three of the five coefficients in eq. (3.17), the entire quartic truncation (including the first subleading term) is necessary for the consistency of the calculation to $\mathcal{O}(q)$ [3].

In Table 1 below, we list the expressions for $S(v)$ for all the elliptic models described in sect. 2 [10]. The one-instanton prepotential for each of these theories is then given in terms of the residue functions defined in eq. (4.2). For $\mathrm{SU}(N)$+ adjoint [18]

$$2\pi i \mathcal{F}_{1-\mathrm{inst}} = \sum_{k=1}^{N} S_k(a_k). \tag{4.4}$$

For $\mathrm{SU}(N) + \begin{smallmatrix}\square\\ \square\end{smallmatrix} + \square\!\square$, $\mathrm{SO}(2N)$+ adjoint, and $\mathrm{SO}(2N+1)$+ adjoint,

$$2\pi i \mathcal{F}_{1-\mathrm{inst}} = \sum_{k=1}^{N} S_k(a_k) - 2S_{m_1}(-\tfrac{1}{2}m_1), \tag{4.5}$$

where m_1 is the mass of the antisymmetric or adjoint hypermultiplet. For $\mathrm{SU}(N) + 2\begin{smallmatrix}\square\\ \square\end{smallmatrix} + 4\,\square$,

$$2\pi i \mathcal{F}_{1-\mathrm{inst}} = \sum_{k=1}^{N} S_k(a_k) - 2S_{m_1}(-\tfrac{1}{2}m_1) - 2S_{m_2}(-\tfrac{1}{2}m_2), \tag{4.6}$$

where m_1 and m_2 are the masses of the antisymmetric hypermultiplets. For $\mathrm{Sp}(2N)$+ adjoint, and $\mathrm{Sp}(2N) + \begin{smallmatrix}\square\\ \square\end{smallmatrix} + 4\,\square$,

$$2\pi i \mathcal{F}_{1-\mathrm{inst}} = -2[\bar{S}_0(0)]^{1/2} \tag{4.7}$$

where

$$S(v) \; = \; \frac{\bar{S}_0(v)}{v^4} \tag{4.8}$$

defines the residue function at the quartic pole at $v = 0$. All of these results have been subjected to a wide variety of consistency checks, as described in ref. [10].

$\mathcal{N} = 2$ gauge theory	$S(v)$
$\mathrm{SU}(N) \; + \; 2 \begin{smallmatrix}\square\\\square\end{smallmatrix} (m_1, m_2) \; + \; 4 \, \square \, (M_j)$	$\dfrac{\prod_{i=1}^N (v{+}a_i{+}m_1) \prod_{i=1}^N (v{+}a_i{+}m_2) \prod_{j=1}^4 (v{+}M_j)}{(v{+}\frac{1}{2}m_1)^2 (v{+}\frac{1}{2}m_2)^2 \prod_{i=1}^N (v{-}a_i)^2}$
$\mathrm{SU}(N) + \begin{smallmatrix}\square\\\square\end{smallmatrix}(m_1) + \square\square(m_2)$	$\dfrac{(v{+}\frac{1}{2}m_2)^2 \prod_{i=1}^N (v{+}a_i{+}m_1) \prod_{i=1}^N (v{+}a_i{+}m_2)}{(v{+}\frac{1}{2}m_1)^2 \prod_{i=1}^N (v{-}a_i)^2}$
$\mathrm{SU}(N) + \mathrm{adjoint}(m)$	$\dfrac{\prod_{i=1}^N [(v{-}a_i)^2 {-} m^2]}{\prod_{i=1}^N (v{-}a_i)^2}$
$\mathrm{SO}(2N) + \mathrm{adjoint}(m)$	$\dfrac{v^4 \prod_{i=1}^N [(v{-}m)^2 {-} a_i^2] \prod_{i=1}^N [(v{+}m)^2 {-} a_i^2]}{(v{+}\frac{1}{2}m)^2 (v{-}\frac{1}{2}m)^2 \prod_{i=1}^N (v^2 {-} a_i^2)^2}$
$\mathrm{SO}(2N+1) + \mathrm{adjoint}(m)$	$\dfrac{v^2 (v{+}m)(v{-}m) \prod_{i=1}^N [(v{-}m)^2 {-} a_i^2] \prod_{i=1}^N [(v{+}m)^2 {-} a_i^2]}{(v{+}\frac{1}{2}m)^2 (v{-}\frac{1}{2}m)^2 \prod_{i=1}^N (v^2 {-} a_i^2)^2}$
$\mathrm{Sp}(2N) + \mathrm{adjoint}(m)$	$\dfrac{(v{+}\frac{1}{2}m)^2 (v{-}\frac{1}{2}m)^2 \prod_{i=1}^N [(v{-}m)^2 {-} a_i^2] \prod_{i=1}^N [(v{+}m)^2 {-} a_i^2]}{v^4 \prod_{i=1}^N (v^2 {-} a_i^2)^2}$
$\mathrm{Sp}(2N) \; + \; \begin{smallmatrix}\square\\\square\end{smallmatrix}(m) \; + \; 4 \, \square (M_j)$	$\dfrac{\prod_{i=1}^N [(v{-}m)^2 {-} a_i^2] \prod_{i=1}^N [(v{+}m)^2 {-} a_i^2] \prod_{j=1}^4 (v^2 {-} M_j^2)}{v^4 (v{+}\frac{1}{2}m)^2 (v{-}\frac{1}{2}m)^2 \prod_{i=1}^N (v^2 {-} a_i^2)^2}$

Table 1

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